CDE/ECE Faculty Initiative Project Instructional Guide


Unit 6
Mathematics Domain
Focus Statement
Students reflect on their own early mathematics experiences by answering a series of questions.

Getting Ready for the Unit
The mathematics domain in the California Preschool Curriculum Framework, Volume 1 (PCF, V1) builds on the mathematics domain in the California Preschool Learning Foundations, Volume 1 (PLF, V1) and is organized by the same strands and substrands. It should be noted that the wording of the substrands in the PCF, V1 is different from that in the PLF, V1. The ten mathematics guiding principles and the environments and materials for the mathematics domain in the PCF, V1 are at the domain level while the vignettes, teachable moments, and interactions and strategies are at the substrand level.

When discussing the relationship between the mathematics domains in the PLF, V1 and the PCF, V1, it is important to guide students in understanding that the foundations are goal-like statements of what we would like to see in young children’s mathematical development and learning while the curriculum framework provides tools for how to support children’s mathematical development and learning. This distinction is addressed in the first key topic of this unit, but it would be helpful to remind students about it as they work in other key topics.

As stated in the introduction to the chapter on mathematics of the PCF, V1, “Young children seem to have an innate sense of informal mathematics. They develop a substantive body of informal knowledge of mathematics from infancy throughout the preschool years” (PCF, V1, p. 232). Also, “[Preschool teachers] help children build their knowledge and skills of mathematics over time, by providing a mathematically rich environment, by modeling mathematical thinking and reasoning, and by introducing children to the language of mathematics (PCF, V1, p. 233). This unit is designed to support students in fulfilling those roles by familiarizing them with the contents of Chapter 6 of the PCF, V1.

Because the mathematical knowledge and skills related to number, quantity, size, shape, and space and the process of mathematical reasoning may be unfamiliar to
some students, it is very important that instructors be very familiar with the mathematics foundations. Then they will be able to provide opportunities for students to develop a basic understanding of these mathematics concepts and skills through lectures or other active learning methods. Descriptions of the mathematics strands can be found in both the PLF, V1 (pp. 145-147) and the PCF, V1 (pp. 241, 259, 272, 281, and 290). Examples of the research base are also in the PLF, V1, (pp. 160-166) and PCF, V1 (pp. 251, 264, and 273). A set of PowerPoint slides in the introduction to the Instructional Guide for the California Preschool Learning Foundations, Volume 1 (PLF, V1)—Mathematics Domain—also provides an overview of the mathematics foundations.

You may want to review “Piecing Together the Mathematics Domain Puzzle” (Activity 4 of the Instructional Guide for the PLF, V1, Mathematics Domain) and “Understanding the Research and Evidence Base for the Mathematics Domain” (Activity 5 of the Instructional Guide for the PLF, V1, Mathematics Domain) as methods for students to familiarize themselves with the mathematics strands, substrands, and foundations.

Some of the key topics in this unit suggest having students do observations in early care and education settings. Therefore, it may be helpful to have a list of programs that students can visit. It will also be helpful to have photographs or video clips of classrooms to use during class sessions, especially if it may be difficult for students to visit programs.

Because mathematics vocabulary is a major part of children’s construction of mathematical concepts, students also need to be very aware of ways to support children who are English learners in understanding this vocabulary. Information in Unit 5 on English-language development in this PCF, V1 instructional guide can help students identify strategies to use with these children.

Motivator and Connection to Experience

Before You Start
Research indicates that understanding mathematics, beyond the intuitive and informal experiences that many children have, is critical to future academic success. However, many students and practitioners may not fully engage young children with mathematics experiences because they are uncertain about their own knowledge and skills relating to mathematics. Consequently, many students find it hard to engage with the concepts and possibilities of helping young children’s development relating to mathematical concepts.

This motivator exercise is adapted from Activity 1 from the Instructional Guide for the California Preschool Learning Foundations, Volume 1—Mathematics Domain—
“Exploring Our Early Connections to the Mathematics Foundations.”

Students are asked to reflect on their own early mathematics experiences by answering a series of questions. As with any activity in which students are asked to reflect on past experiences, some strong emotional responses may be elicited, and there may be students whose memories will not be positive ones. It is important to acknowledge this and allow students to participate in the discussion as much or as little as they choose. It is also recommended that a short break be planned after this activity so that there is an opportunity for students to deal with any strong emotions that may have surfaced.

Active Learning  Getting it started
Explain that you are going to ask the students to think about their early experiences learning mathematics by answering the following questions:

• When you reflect on your own experiences learning mathematics, what specific memories come to mind? You may remember particular classes, instructors, activities, or homework assignments.

• Think about a time when you enjoyed mathematics. Why do you think you enjoyed mathematics? What do you remember that made mathematics enjoyable? Again, consider particular classes, instructors, activities, or homework that may have been particularly interesting, fun, or rewarding.

• Was there a time when you thought mathematics was hard or you couldn’t understand it? Why do you think this happened? Were there particular classes, instructors, activities, or homework that may have posed a challenge for you?

Keeping it going
Have students find one or two partners and take turns sharing their responses. Remind students that they can share as much or as little as they are comfortable doing. Then ask for volunteers to share some of their memories and stories with the whole class.

Putting it together
Conclude with a class discussion about the experiences that
were shared. The following questions may help guide the discussion:

- What are some things that stand out for you from people’s remembrances and stories?
- Which ones resonated with you? Which ones surprised you?
- What are some themes that emerged from these experiences?
- Why is it helpful to think about the ways you learned mathematics and how you feel about math?
- What do you want to remember from this exercise when you are planning curriculum to support young children’s mathematical development?
When you reflect on your own experiences learning mathematics, what specific memories come to mind?

Think about a time when you enjoyed mathematics. Why do you think you enjoyed it?

Was there a time when you thought mathematics was hard or you couldn’t understand it?
Mathematics

• What are some things that stand out for you from people’s remembrances and stories?
• Which ones resonated with you? Which ones surprised you?
• What are some themes that emerged from these experiences?

Mathematics

• Why is it helpful to think about the ways you learned mathematics and how you feel about math?
• What do you want to remember from this exercise when you are planning curriculum to support young children’s mathematical development?
Focus Statement
Students explore the rationale for the mathematics domain and become familiar with the domain’s organization and guiding principles.

Before You Start
There are three major content areas or subtopics for this key topic: rationale for the domain, organization of the domain, and guiding principles for the domain. These three subtopics each have a set of active learning experiences that are designed to support students in acquiring an overview of the domain’s content while learning where they can locate this content for future use in their work.

If students have already done Key Topic 1 in Unit 1 of this instructional guide that covers the curriculum framework’s overall organization or have explored the organizational structure of one of the four domains, they will already be fairly familiar with how each domain is structured. In that case, you may choose to do a review of the structure of the mathematics domain and skip the content on the domain structure as you go through this key topic. Additionally, the tabbing in the subtopic below on “Organization of the Mathematics Domain” is very similar to the tabbing in Key Topic 1, Unit 1.

The California Preschool Curriculum Framework, Volume 1 (PCF, V1) contains questions for reflection at the end of each strand in the mathematics domain. These questions focus on teachers’ practices and experiences—either of those in the vignette or of the students. The reflection questions in this key topic are to help students think about how the overall content and structure of the mathematics domain in the PCF, V1 and how this knowledge of the domain might be useful in their work.

The following are suggested considerations for one of the active learning experiences:

Subtopic 2: Organization of the Mathematics Domain
In the “Getting it started” segment, small notes or other markers (e.g., Post-it® notes, flags, page markers) are suggested to be used as tabs for the PCF, V1. In the “Taking it further” segment, students are asked to create an organizational chart of the domain. Materials could include easel pad sheets and colored markers.
The following content can be found in the *California Preschool Learning Foundations, Volume 1* (PLF, V1) and the *California Preschool Curriculum Framework, Volume 1* (PCF, V1) and could be used as part of an introduction or review of mathematics for students when they begin this unit.

**California Preschool Learning Foundations, Volume 1:**

- Discussion of the mathematics strands (pp. 145-147)
- Review of the research for each strand (pp. 160-166)
- Glossary related to mathematics (p. 167)

**California Preschool Curriculum Framework, Volume 1:**

- Discussion of the mathematics strands (pp. 241, 259, 272, 281, and 290)
- Research highlights (pp. 251, 264, and 273)
- Glossary related to mathematics (p. 326)

This content could be delivered through lectures and/or assigned readings.

Depending on students’ experience with the mathematics domain in the *California Preschool Learning Foundations, Volume 1* or with early mathematics in general, you may want to review the mathematics strands and substrands as part of the introduction to this key topic. The handout following this key topic may be used as a reference. A description of the five strands can be found on pages 239-240 of the *California Preschool Curriculum Framework, Volume 1* and are listed here for reference:

- Number Sense
- Algebra and Functions (Classification and Patterning)
- Measurement
- Geometry
- Mathematical Reasoning
Active Learning

**Subtopic 1: Rationale for the Mathematics Domain**

**Getting it started**
Begin by having students read the introduction to the mathematics domain in the PCF, V1 on pages 232-233. Ask them to underline any words, phrases, or sentences that stand out for them.

Next have students review their underlined words and note which ones represented familiar concepts or knowledge, which ones were new, and which ones particularly intrigued them.

**Keeping it going**
Continue the exercise by having students work individually, in pairs, or in small groups to make a list of the key points about young children’s development of mathematical knowledge and skills and how mathematics is part of a preschool program. Then have students individually write a one- or two-paragraph summary of why mathematics learning is an important part of a preschool program.

**Putting it together**
Have students form pairs and share their summaries with each other. Then conclude this subtopic with a full class discussion around the following questions:

- What were some of the main concepts or points in your summaries?
- Which ones were similar to your partner’s? Which ones were different?
- What insights about planning mathematical learning environments and experiences are starting to emerge for you? What questions or concerns?
- What will you want to keep in mind about young children and mathematics?

**Subtopic 2: Organization of the Mathematics Domain**

**Getting it started**
Have students become familiar with the organization of the mathematics domain by reading or reviewing the “Organization
of the Framework” on pages 9-10 of the PCF, V1 and then tabbing the following elements of the domain:

- Domain Guiding Principles (pp. 233-236)
- Environments and Materials (pp. 237-239)
- Summary of the Strands and Substrands (pp. 239-240)
- Strands (pp. 241, 259, 272, 281, and 290)
- Substrands (pp. 242, 251, 260, 264, 273, 282, 286, and 291)
- Research Highlights (pp. 251, 264, and 273)
- Engaging Families (pp. 257-258, 269-270, 279-280, 288-289, and 294)
- Questions for Reflection (pp. 258, 271, 280, 289, and 294)

Next ask the students to review the summary of the mathematics strands and substrands on pages 239-240 of the California Preschool Curriculum Framework, Volume 1 (PCF, V1) and the appendix of the California Preschool Learning Foundations, Volume 1 (PLF, V1) on pages 189-192, which is a summary of the strands, substrands, and foundations for the mathematics domain. The PLF, V1 appendix for this domain can also be found at the end of this key topic in Handout 1. Have students compare the two summaries so that they see the parallel organization of the PCF, V1 and the PLF, V1.

Be sure that students note that the names of the strands are identical but that the descriptions of the substrands in the PCF, V1 and the wording of the substrands in the PLF, V1 are not the same. Also, the labels of the substrands in the PCF, V1 are not identical to the wording of the substrands in the PLF, V1. For example, the first substrand under the number sense strand in the PCF, V1 is “Understanding Number and Quantity” while the wording for the same substrand in the PLF, V1 is “Children begin to understand numbers and quantities in their everyday environment” for children around 48 months of age and “Children expand their understanding of numbers and quantities in their everyday environment” for children around 60 months of age.
It is important for students to understand that the PLF, V1 describes what children learn and the PCF, V1 describes how teachers can plan learning experiences. Emphasize that this parallel structure does not mean that teachers are to apply the curriculum framework in a one-to-one match to the foundations.

**Keeping it going**
Have students continue identifying the structure of the mathematics domain by tabbing the following elements:

- Vignettes and teachable moments for each substrand
- Interactions and strategies for each substrand (point out that these follow each vignette)

You may also ask students to note that there are “Teacher Resources” on page 297 of the PCF, V1.

**Taking it further**
Have students continue to explore the structure of the mathematics domain by creating an organizational chart of the major elements.

Students can create this chart individually or in pairs or small groups. Assign each student or group of students a strand. Because some strands have one substrand and others have two, you may wish to assign two groups to the strands that have two substrands. Show a sample for part of a strand and substrand to demonstrate the shapes and symbols to use in the charts. Because all the charts will eventually be posted to show the organization of the whole domain, it is helpful if the groups use the same organizing labels.

**Putting it together**
Have the students display their charts in the order of the strands and substrands. After all the students have had a chance to look at all the charts, discuss the following questions as a whole class:

- What stood out for you when you looked at organizational charts of the mathematics domain?
- What surprised you? Which substrand had elements that were the most familiar to you? The most unfamiliar?
- What patterns did you see in the organizational structure of
the domain?

- How might you use this organizational chart of the domain as a reference for your work in planning mathematical environments and learning experiences for children?

**Subtopic 3: Guiding Principles for the Mathematics Domain**

**Getting it started**
Ask students to read the 10 mathematics domain guiding principles (PCF, V1, pp. 233-236) and have them note some key words or phrases for each domain principle.

**Keeping it going**
Start with the first domain guiding principle and ask one student to read the principle and share one of the key words or phrases. Then have other students share other key words or phrases until the students’ lists are exhausted for that domain principle. Repeat this process with the other nine domain principles.

**Putting it together**
Conclude the discussion of the 10 mathematics guiding principles with the following questions:

- Which domain principle(s) stood out for you?
- Which ones were familiar and consistent with what you already know about young children and mathematics? Which ones were new to you?
- Why do you think these domain principles are relevant in planning curriculum to support children’s mathematics knowledge and skills?
- Which domain principle do you want to learn more about? How could you do that?

**Reflection**
There are two sets of reflective questions offered for this key topic. The first specifically addresses the rationale, organizational structure, and guiding principles of the mathematics domain:

- In thinking about the rationale, guiding principles, and organization of the mathematics domain of the PCF, V1,
what concepts or information stood out for you?

• What was clear? What might have been confusing?

• What was useful in helping you understand the overall content and structure of this domain?

• How might you apply this understanding in your current or future work? What additional supports might you need?

The second set of questions is more general and can be used in this and other key topics:

• What ideas stood out most for you today?

• Which ones reinforced what you have already learned or experienced? Which ones gave you a new perspective or insight?

• How might you apply a new perspective to your work now or in the future?

• What further information or support do you need?

• What first step do you need to take?

**Deeper Understanding**

Remind students that the joint position statement by the National Association for the Education of Young Children and the National Council of Teachers of Mathematics, “Early Childhood Mathematics: Promoting Good Beginnings,” was cited as one of the sources for the 10 mathematics guiding principles. The position statement can be downloaded from http://www.naeyc.org/positionstatements/mathematics. Have students read the full position statement and write a short paper that includes responses to the following questions:

• What are the key points in this position statement?

• Where do you see similarities between the position statement recommendations and the 10 guiding principles in the mathematics domain of the PCF, V1? Where are there differences?

• What additional clarification or information regarding mathematics and young children did you learn from this
position statement?

- What similarities did you see between the learning paths in the position statement and the mathematics strands and substrands of the PLF, V1 or PCF, V1?

- What key messages from this position statement can you apply in your current or future work with young children?
Mathematics

Number Sense

<table>
<thead>
<tr>
<th>At around 48 months of age</th>
<th>At around 60 months of age</th>
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</thead>
<tbody>
<tr>
<td><strong>1.0 Children begin to understand numbers and quantities in their everyday environment.</strong></td>
<td><strong>1.0 Children expand their understanding of numbers and quantities in their everyday environment.</strong></td>
</tr>
<tr>
<td>1.1 Recite numbers in order to ten with increasing accuracy.</td>
<td>1.1 Recite numbers in order to twenty with increasing accuracy.</td>
</tr>
<tr>
<td>1.2 Begin to recognize and name a few written numerals.</td>
<td>1.2 Recognize and know the name of some written numerals.</td>
</tr>
<tr>
<td>1.3 Identify, without counting, the number of objects in a collection of up to three objects (i.e., subitize).</td>
<td>1.3 Identify, without counting, the number of objects in a collection of up to four objects (i.e., subitize).</td>
</tr>
<tr>
<td>1.4 Count up to five objects, using one-to-one correspondence (one object for each number word) with increasing accuracy.</td>
<td>1.4 Count up to ten objects, using one-to-one correspondence (one object for each number word) with increasing accuracy.</td>
</tr>
<tr>
<td>1.5 Use the number name of the last object counted to answer the question, “How many . . . ?”</td>
<td>1.5 Understand, when counting, that the number name of the last object counted represents the total number of objects in the group (i.e., cardinality).</td>
</tr>
</tbody>
</table>

**2.0 Children begin to understand number relationships and operations in their everyday environment.**

| 2.1 Compare visually (with or without counting) two groups of objects that are obviously equal or nonequal and communicate, “more” or “same.” | 2.1 Compare, by counting or matching, two groups of up to five objects and communicate, “more,” “same as,” or “fewer” (or “less”). |
| 2.2 Understand that adding to (or taking away) one or more objects from a group will increase (or decrease) the number of objects in the group. | 2.2 Understand that adding one or taking away one changes the number in a small group of objects by exactly one. |
### Mathematics

<table>
<thead>
<tr>
<th>At around 48 months of age</th>
<th>At around 60 months of age</th>
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</thead>
<tbody>
<tr>
<td><strong>2.3</strong> Understand that putting two groups of objects together will make a bigger group.</td>
<td><strong>2.3</strong> Understand that putting two groups of objects together will make a bigger group and that a group of objects can be taken apart into smaller groups.</td>
</tr>
<tr>
<td><strong>2.4</strong> Solve simple addition and subtraction problems nonverbally (and often verbally) with a very small number of objects (sums up to 4 or 5).</td>
<td><strong>2.4</strong> Solve simple addition and subtraction problems with a small number of objects (sums up to 10), usually by counting.</td>
</tr>
</tbody>
</table>

### Algebra and Functions

(Classification and Patterning)

<table>
<thead>
<tr>
<th>At around 48 months of age</th>
<th>At around 60 months of age</th>
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</thead>
<tbody>
<tr>
<td><strong>1.0</strong> Children begin to sort and classify objects in their everyday environment.</td>
<td><strong>1.0</strong> Children expand their understanding of sorting and classifying objects in their everyday environment.</td>
</tr>
<tr>
<td><strong>1.1</strong> Sort and classify objects by one attribute into two or more groups, with increasing accuracy.</td>
<td><strong>1.1</strong> Sort and classify objects by one or more attributes, into two or more groups, with increasing accuracy (e.g., may sort first by one attribute and then by another attribute).</td>
</tr>
<tr>
<td><strong>2.0</strong> Children begin to recognize simple, repeating patterns.</td>
<td><strong>2.0</strong> Children expand their understanding of simple, repeating patterns.</td>
</tr>
<tr>
<td><strong>2.1</strong> Begin to identify or recognize a simple repeating pattern.</td>
<td><strong>2.1</strong> Recognize and duplicate simple repeating patterns.</td>
</tr>
<tr>
<td><strong>2.2</strong> Attempt to create a simple repeating pattern or participate in making one.</td>
<td><strong>2.2</strong> Begin to extend and create simple repeating patterns.</td>
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</tbody>
</table>
### Measurement

<table>
<thead>
<tr>
<th>At around 48 months of age</th>
<th>At around 60 months of age</th>
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<tbody>
<tr>
<td><strong>1.0</strong> Children begin to compare and order objects.</td>
<td><strong>1.0</strong> Children expand their understanding of comparing, ordering, and measuring objects.</td>
</tr>
<tr>
<td>1.1 Demonstrate awareness that objects can be compared by length, weight, or capacity, by noting gross differences, using words such as bigger, longer, heavier, or taller, or by placing objects side by side to compare length.</td>
<td>1.1 Compare two objects by length, weight, or capacity directly (e.g., putting objects side by side) or indirectly (e.g., using a third object).</td>
</tr>
<tr>
<td>1.2 Order three objects by size.</td>
<td>1.2 Order four or more objects by size.</td>
</tr>
<tr>
<td>1.3 Measure length using multiple duplicates of the same-size concrete units laid end to end.</td>
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</table>

### Geometry

<table>
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<tr>
<th>At around 48 months of age</th>
<th>At around 60 months of age</th>
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</thead>
<tbody>
<tr>
<td><strong>1.0</strong> Children begin to identify and use common shapes in their everyday environment.</td>
<td><strong>1.0</strong> Children identify and use a variety of shapes in their everyday environment.</td>
</tr>
<tr>
<td>1.1 Identify simple two-dimensional shapes, such as a circle and square.</td>
<td>1.1 Identify, describe, and construct a variety of different shapes, including variations of a circle, triangle, rectangle, square, and other shapes.</td>
</tr>
<tr>
<td>1.2 Use individual shapes to represent different elements of a picture or design.</td>
<td>1.2 Combine different shapes to create a picture or design.</td>
</tr>
<tr>
<td><strong>2.0</strong> Children begin to understand positions in space.</td>
<td><strong>2.0</strong> Children expand their understanding of positions in space.</td>
</tr>
<tr>
<td>2.1 Identify positions of objects and people in space, such as in/on/under, up/down, and inside/outside.</td>
<td>2.1 Identify positions of objects and people in space, including in/on/under, up/down, inside/outside, beside/between, and in front/behind.</td>
</tr>
</tbody>
</table>
**Mathematical Reasoning**

<table>
<thead>
<tr>
<th></th>
<th>At around 48 months of age</th>
<th>At around 60 months of age</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.0</strong> Children use mathematical thinking to solve problems that arise in their everyday environment.</td>
<td>1.0 Children expand the use of mathematical thinking to solve problems that arise in their everyday environment.</td>
<td></td>
</tr>
<tr>
<td><strong>1.1</strong> Begin to apply simple mathematical strategies to solve problems in their environment.</td>
<td>1.1 Identify and apply a variety of mathematical strategies to solve problems in their environment.</td>
<td></td>
</tr>
</tbody>
</table>
Organization and Rationale of the Mathematics Domain

Mathematics Domain Strands

**Number Sense**
1.0 Understanding Number and Quantity
2.0 Understanding Number Relationships and Operations

**Algebra and Functions** (Classification and Patterning)
1.0 Classification
2.0 Patterning
**Mathematics Domain Strands**

**Measurement**
1.0 Compare, Order, and Measure Objects

**Geometry**
1.0 Shapes
2.0 Positions in Space

**Mathematical Reasoning**
1.0 Promoting Mathematical Reasoning and Problem Solving

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**Rationale for the Mathematics Domain**

- What were some of the main concepts or points in your summaries?
- Which ones were similar to your partner’s? Which ones were different?
- What insights about planning mathematical learning environments and experiences are starting to emerge for you? What questions or concerns?
- What will you want to keep in mind about young children and mathematics?
Organization of the Mathematics Domain

- Domain Guiding Principles
- Environments and Materials
- Summary of the Strands and Substrands
- Strands
- Substrands
- Research Highlights
- Engaging Families
- Questions for Reflection

Organization of the Mathematics Domain

- Vignettes
- Teachable moments
- Interactions and strategies
Organization of the Mathematics Domain

- What stood out for you when you looked at organizational charts of the mathematics domain?
- What surprised you? Which substrand had elements that were the most familiar to you? The most unfamiliar?
- What patterns did you see in the organizational structure of the domain?
- How might you use this organizational chart of the domain as a reference for your work in planning mathematical environments and learning experiences for children?

Guiding Principles for the Mathematics Domain

- Build on preschool children’s natural interest in mathematics and their intuitive and informal mathematical knowledge
- Encourage inquiry and exploration to foster problem solving and mathematical reasoning
- Use everyday activities as natural vehicles for developing preschool children’s mathematical knowledge
Guiding Principles for the Mathematics Domain

- Introduce mathematical concepts through intentionally planned experiences
- Provide a mathematically rich environment
- Provide an environment rich in language, and introduce preschool children to the language of mathematics

Guiding Principles for the Mathematics Domain

- Support English learners in developing mathematical knowledge as they concurrently acquire English
- Observe preschool children and listen to them
- Recognize and support the individual
- Establish a partnership with parents and other caregivers in supporting children’s learning of mathematics
Guiding Principles for the Mathematics Domain

- Which domain principle(s) stood out for you?
- Which ones were familiar and consistent with what you already know about young children and mathematics? Which ones were new to you?
- Why do you think these principles are relevant in planning curriculum to support children’s mathematics knowledge and skills?
- Which principle do you want to learn more about? How could you do that?

Rationale, Guiding Principles, and Organization of the Mathematics Domain

- What concepts or information stood out for you?
- What was clear? What might have been confusing?
- What was useful in helping you understand the overall content and structure of this domain?
- How might you apply this understanding in your current or future work? What additional supports might you need?
• What ideas stood out most for you today?
• Which ones reinforced what you have already learned or experienced? Which ones gave you a new perspective or insight?
• How might you apply a new perspective to your work now or in the future?
• What further information or support do you need?
• What first step do you need to take?
Focus Statement
Students become familiar with environmental arrangements and materials that can be used to support children’s development and learning in mathematics.

Before You Start
The following are suggested considerations for one of the active learning segments found later in this key topic:

• **Putting it together**: In this segment, students review photographs of preschool classrooms to identify examples of environments and materials that support children’s mathematical development. Video clips could also be used, but photographs might be easier for students who do not have a lot of background or experience in early mathematics.

Information Delivery
The following content from the *California Preschool Curriculum Framework, Volume 1* (PCF, V1) could be used as part of an introduction to or review of mathematics for students when they begin this key topic:

• Overview of how young children construct mathematical knowledge from infancy through their preschool years (pp. 232-233)

• Importance of the teacher’s role in everyday classroom practices, including helping “children build their knowledge and skills of mathematics over time by providing a mathematically rich environment, by modeling mathematical thinking and reasoning, and by introducing children to the language of math” (p. 233)

• Examples of how the physical environment and materials support children’s active construction of mathematical concepts (pp. 232-233)
Active Learning

Getting it started
Begin by having students review the environments and materials section in the mathematics domain (PCF, V1, pp. 237-239). Explain that there are six major strategies related to environments and materials listed in bold-faced type and that each strategy includes some descriptors and examples. Ask the students to imagine that they are setting up a new preschool classroom. Which strategy should be uppermost in their minds?

Keeping it going
Ask students to group themselves according to the strategy they selected. Explain that the students have 15 minutes to prepare a five-minute presentation explaining how their choice supports children’s mathematical development. They are to include concrete examples of what that strategy would look like in a classroom.

If one or more of the strategies were not selected by any students, have a full class discussion about how children’s mathematical knowledge and skills are fostered by the strategy and some examples of the strategy.

Taking it further
Have students review the mathematics strands and substrands and list them on one axis of a grid. Then have them write the environments and materials on the other axis and think about which of those might support different substrands. A sample grid is provided following this key topic. Students can then place an “X” on the grid where they see a connection. Make sure students understand that they may see multiple connections for a specific environment or materials. Students can work on the grids individually or in groups.

Review the completed grids with the whole class. As students identify where they saw a substrand supported by a specific environment or material, ask them to explain why.

Putting it together
Show students photographs of different areas of preschool classrooms and ask them to identify examples of any of the mathematical environments and materials described in the PCF, V1. You may choose to show the photographs to the entire class and have students comment on all of them, or you
may provide specific photographs to groups of students to work on together.

Have students summarize examinations of the photographs with the following points:

- Descriptions of the examples of the environmental arrangements or materials they observed
- How these examples demonstrated planning by the teacher
- How these examples promoted children’s development of mathematical knowledge and skills
- One or two suggestions for other environments or materials

---

**Reflection**

There are two sets of reflective questions offered for this key topic. The first specifically addresses the environment and materials in the mathematics domain:

- When thinking about planning the environment and materials for a preschool program to support children’s development of mathematical knowledge and skills, which ideas stood out for you?
- Which ones seem to be easier to implement? Which ones might pose more challenges?
- What considerations should you keep in mind for ensuring that the environment and materials support children who are learning English or have disabilities to progress in their mathematical development?
- Where might you find additional support in setting up the environment and materials in a classroom to foster children’s mathematical learning?

The second set of questions is more general and can be used in this and other key topics:

- What ideas stood out most for you today?
- Which ones reinforced what you have already learned or experienced? Which ones gave you a new perspective or insight?
• How might you apply a new perspective to your work now or in the future?

• What further information or support do you need?

• What first step do you need to take?

---

**Deeper Understanding**

Discuss with students the importance of ensuring that all children in their classrooms have access to mathematics learning experiences and the critical role that language plays in children’s development of a conceptual framework of mathematics. Have students write a short paper on what teachers should plan in the environment and materials to support children who are English learners. Students may use two of the mathematics guiding principles as starting points (i.e., “Provide an environment rich in language, and introduce preschool children to the language of mathematics” and “Support English learners in developing mathematical knowledge as they concurrently acquire English,” PCF, V1, p. 235).

Students might also use one or more of the following resources:

• “Teacher Resources” (PCF, V1, p. 297)

• “References” (PCF, V1, pp. 298-299)

• Interviews with preschool teachers

• The resource guide *Preschool English Learners: Principles and Practices to Promote Language, Literacy, and Learning (Second Edition)*
### Environments and Materials →

<table>
<thead>
<tr>
<th>Strands and Substrands</th>
<th>Enrich the environment with objects and materials that promote mathematical growth</th>
<th>Integrate math-related materials in all interest areas</th>
<th>Provide real-life settings to investigate</th>
<th>Use materials and objects that are relevant and meaningful to the children in your group</th>
<th>Use children's books to explore mathematics with children</th>
<th>Be intentional and mindful in setting up and using the physical environment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strand: Number Sense</strong></td>
<td></td>
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<tr>
<td>1.0 Understanding Number and Quantity</td>
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<tr>
<td>2.0 Understanding Number Relationships and Operations</td>
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<tr>
<td><strong>Strand: Algebra and Functions</strong></td>
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<td></td>
</tr>
<tr>
<td>1.0 Classification</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2.0 Patterning</td>
<td></td>
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<tr>
<td><strong>Strand: Measurement</strong></td>
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<td></td>
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<tr>
<td>1.0 Compare, Order, and Measure Objects</td>
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<tr>
<td><strong>Strand: Geometry</strong></td>
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<tr>
<td>1.0 Shapes</td>
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<td></td>
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<tr>
<td>2.0 Positions in Space</td>
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<td></td>
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<tr>
<td><strong>Strand: Mathematical Reasoning</strong></td>
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<td></td>
</tr>
<tr>
<td>1.0 Promoting Mathematical Reasoning and Problem Solving</td>
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</tr>
</tbody>
</table>
Environments and Materials That Support Mathematics

- Descriptions of the examples of the environmental arrangements or materials they observed
- How these examples demonstrated planning by the teacher
- How these examples promoted children’s development of mathematical knowledge and skills
- One or two suggestions for other environments or materials
Environments and Materials That Support Mathematics

• When thinking about planning the environment and materials, which ideas stood out for you?
• Which ones seem to be easier to implement? Which ones might pose more challenges?

Environments and Materials That Support Mathematics

• What considerations should you keep in mind for ensuring that the environment and materials support children who are learning English or have disabilities to progress in their mathematical development?
• Where might you find additional support in setting up the environment and materials in a classroom to foster children’s mathematical learning?
• What ideas stood out most for you today?
• Which ones reinforced what you have already learned or experienced? Which ones gave you a new perspective or insight?
• How might you apply a new perspective to your work now or in the future?
• What further information or support do you need?
• What first step do you need to take?
Focus Statement
Students become familiar with the recommended interactions and strategies in the mathematics domain by reviewing vignettes for each substrand and the interactions and strategies that are illustrated in them.

Before You Start
There are 48 interactions and strategies in Chapter 6 of the California Preschool Curriculum Framework, Volume 1 (PCF, V1), and these interactions and strategies are listed after the vignettes presented in each of the eight substrands. For each vignette, a teachable moment or an opportunity to plan learning is identified followed by some interactions and strategies that relate to the vignette. If students are just beginning their study of the PCF, V1 with this key topic, it may be helpful to do a review of the domain’s organizational structure first. You could use the “Getting it started” segment in Key Topic 1 of this Unit 6, “Organization and Rationale of the Mathematics Domain,” or reference Unit 1, Key Topic 1 of this instructional guide.

Students initially learn about the interactions and strategies through reviewing the vignettes and becoming familiar with the suggested interactions and strategies. Next they reflect on the kinds of interactions and strategies suggested as well as the emphasis on mathematics vocabulary before being asked to identify examples of the interactions and strategies in early care and education settings. Suggested methods for identifying these examples are classroom observations, video clips, or presentations by teachers.

If you plan to have students do classroom observations, you may need to remind them about protocols for conducting these observations. Also, remind them that what they will observe in one visit is a snapshot and not a comprehensive documentary of the program. It will not be possible to see examples of all strategies and interactions in a single observation.

Table 4 indicates how many interactions and strategies there are for each strand and substrand. This may help you determine the student groupings and assignments for the active learning.
### Table 4. Total number of vignettes, interactions, and strategies for the mathematics domain by strand and substrand

<table>
<thead>
<tr>
<th>Strands and substrands</th>
<th>Number of vignettes</th>
<th>Number of interactions &amp; strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strand: Number Sense</strong> <em>(PCF, V1, pp. 241-258)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 Understanding Number and Quantity</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>2.0 Understanding Number Relationships and Operations</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td><strong>Strand: Algebra and Functions</strong> <em>(Classification and Patterning)</em> <em>(PCV, V1, pp. 259-271)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 Classification</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2.0 Patterning</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Strand: Measurement</strong> <em>(PCF, V1, pp. 272-280)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 Compare, Order, and Measure Objects</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td><strong>Strand: Geometry</strong> <em>(PCF, V1, pp. 281-289)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 Shapes</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2.0 Positions in Space</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td><strong>Strand: Mathematical Reasoning</strong> <em>(PCF, V1, pp. 290-294)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 Promoting Mathematical Reasoning and Problem Solving</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

**Source:** *California Preschool Curriculum Framework, Volume 1.*

**Note to Faculty:** A full list of the interactions and strategies is on pages 206-211 at the end of this key topic.
Information Delivery

The following content can be found in the *California Preschool Learning Foundations, Volume 1* (PLF, V1) and the *California Preschool Curriculum Framework, Volume 1* (PCF, V1) and could be used as part of an introduction to or review of mathematics and young children as students begin work in this key topic.

**California Preschool Learning Foundations, Volume 1:**

- Discussion of the mathematics strands (pp. 145-147)
- Review of the research for each strand (pp. 160-166)
- Glossary related to mathematics (p. 167)

**California Preschool Curriculum Framework, Volume 1:**

- Brief overview of how young children construct mathematical knowledge and the teacher’s role (pp. 232-233)
- Discussion of the mathematics strands (pp. 241, 259, 272, 281, and 290)
- Research highlights (pp. 251, 264, and 273)
- Glossary related to mathematics (p. 326)

Information about the organizational structure of the PCF, V1 can be found in these places in the PCF, V1:

- Review of the organizational structure of the PCF, V1 (pp. 9-10)
- Unit 6, Key Topic 1, “Organization and Rationale of the Mathematics Domain,” in this instructional guide

This content could be delivered through lectures and/or assigned readings.

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Active Learning

**Getting it started**

It may be helpful to begin the segment by reviewing with students the organizational structure of the PCF, V1. The extent of this review will depend on students’ earlier work in identifying
the underlying structure of each domain of the PCF, V1.

Key Topic 1 in Units 1, 3, 4, and 5 all include opportunities for students to familiarize themselves with this organizational structure.

Explain that this key topic focuses on the interactions and strategies that support children’s mathematical development. Be sure the students understand that each substrand contains vignettes that are followed by teachable moments or opportunities to plan learning and interactions and strategies related to the vignette.

Depending on the number of students in your class, you may choose to have the students work individually, in pairs, or in small groups.

Have students read the vignette(s) and teachable moments in their assigned substrand and the interactions and strategies described for each vignette. Have them consider the following questions as they read the vignettes and interactions and strategies:

• Which interactions or strategies did the teacher in the vignette use? How were they used?

• Which interactions or strategies were not observed? How could they be included?

Depending on the students’ prior knowledge of how young children construct mathematical knowledge, you may choose to have students discuss some of their responses as a large group. This will provide students with additional examples of how the mathematics interactions and strategies can be used.

Keeping it going
Next have the students make a list of the interactions and strategies for each substrand. A list of the interactions and strategies is provided as Handout 1 following this key topic. If you have groups of students do the lists for specific substrands, provide a way for students to share and obtain a complete list of the strategies (e.g., post lists online, write each list on chart paper or a whiteboard, type the lists on a computer and show the lists through an LCD projector).
After giving the students time to review their lists and clarify any questions or vocabulary about them, have the students respond to or discuss the following questions:

- What stands out for you from these lists?
- What similarities do you see among the different substrands?
- Do you see any general categories that the strategies could be grouped in? What are they?
- Why do you think there are so many strategies that relate to teaching mathematics vocabulary and having conversations with children?
- How might you keep and organize these interactions and strategies so they could be a resource for you?

**Taking it further**

Seeing some of the interactions and strategies being implemented in preschool programs would be very helpful for students. If there are preschool programs where students can do observation visits, ask them to do so. Explain that they are to look for examples of the interactions and strategies for one or more of the mathematics strands and substrands.

You may choose to have specific students focus their observations on a specific strand or substrand and make sure that all the strands and substrands are assigned. Then during classroom discussions, students will have an opportunity to learn about a broader range of interactions and strategies than they might observe on their own.

Doing the observations in pairs would also provide students with an opportunity to compare and reflect on their observations with a partner. Suggest that students also ask the teachers if they have time to meet after the observation. A conversation with the teacher could provide students with a chance to ask questions and build an increased understanding of what they observed.

Have students prepare summaries of their observations that include the following information:

- Context of the activity (e.g., number of children, number and
role of adults, description of the activity including any materials, where the activity was conducted, length of the activity)

- Observations (e.g., what children did and said, what adults did and said)
- Examples of interactions and strategies observed
- Which examples seemed to be intentionally planned
- Which examples seemed to be teachable moments (i.e., the teacher responded to something a child or group of children did or said)

If it is not feasible for students to do classroom observations, video clips would be an alternative. If you use video clips instead of classroom observations, you can follow the same guidelines in assigning students and substrands.

If observations and video clips are not a possibility, you may consider inviting a teacher or panel of teachers to share examples of mathematical strategies they use in their early care and education settings. It may be helpful to give the speaker(s) a list of the mathematics strategies and interactions if they are not already familiar with the PCF, V1.

**Putting it together**

Whether students summarized observations of actual early care and education settings or video clips, provide time for them to share their findings. The following questions could be used in a class discussion:

- What examples of mathematics interactions or strategies stood out for you?
- Which ones were easier to identify? Which ones were harder? Why?
- Did some strategies or interactions appear more frequently than others? Which ones?
- Did some strategies or interactions appear to be more effective than others? Which ones?
- Were there examples where an interaction or strategy was
specifically designed for a child who is an English learner? Which ones?

- Did you see opportunities for a strategy or interaction to be used where they were not used?

What other strategies or interactions might have been used?

If you invited speakers, these questions could be used to reflect on the presentation:

- What examples shared by the speaker(s) caught your attention or stood out for you?
- Which ones were similar to those in the PCF, V1? Which ones were new to you?
- What did you learn about how the teacher(s) planned what interactions or strategies to use?
- What do you want to remember to use in your current or future work?

Reflection

There are two sets of reflective questions offered for this key topic. The first specifically addresses the interactions and strategies in this key topic:

- What interactions and strategies stand out for you?
- Which interactions and strategies were the least familiar to you? Were they from primarily one substrand or across several substrands?
- What other information do you need to help you better understand the interactions and strategies that were least familiar to you?
- Were there some mathematical-related vocabulary or concepts that were new to you?
- Where might you find resources or support?

The second set of questions is more general and can be used in this and other key topics:

- What ideas stood out most for you today?
• Which ones reinforced what you have already learned or experienced? Which ones gave you a new perspective or insight?

• How might you apply a new perspective to your work now or in the future?

• What further information or support do you need?

• What first step do you need to take?

**Deeper Understanding**

This exercise provides students with an opportunity to read about other mathematical interactions and strategies and critically compare them with those in the PCF, V1. *The Intentional Teacher* (Epstein 2007), also mentioned in Key Topic 3, Unit 3 of this instructional guide, has a chapter on “Mathematics and Scientific Inquiry” that includes several teaching strategies. Students are to read this chapter in *The Intentional Teacher* or another book or article and write a paper that addresses the following questions:

• What were some of the key strategies or kinds of strategies suggested?

• How did they compare with the mathematics strategies in the PCF, V1? In what way were they similar or different?

• What were some new ideas or insights you gained about the kinds of interactions and strategies to support young children’s mathematical development?

• What could you use from this book/article in your current or future work?
### Strand: Number Sense

<table>
<thead>
<tr>
<th>Substrand: Understanding Number and Quantity</th>
<th>Substrand: Understanding Number Relationships and Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observe and listen to children’s counts.</td>
<td>Promote the use of comparison terms (more, same as, fewer, or less) through everyday interactions.</td>
</tr>
<tr>
<td>Encourage counting during everyday interactions and routines.</td>
<td>Use everyday interactions and routines to illustrate and discuss addition and subtraction transformations.</td>
</tr>
<tr>
<td>Include preschool children’s home language in counting activities, whenever possible.</td>
<td>Introduce preschool children to the concepts of addition and subtraction through literature, songs, and games.</td>
</tr>
<tr>
<td>Ask questions that encourage purposeful counting.</td>
<td>Make estimations.</td>
</tr>
<tr>
<td>Foster one-to-one correspondence within the context of daily routines.</td>
<td>Use graphing with children.</td>
</tr>
</tbody>
</table>
| Support preschool children’s ability to apply the counting procedure.  
  • Provide lots of objects to count.  
  • Start with small sets of objects.  
  • Start with objects arranged linearly  
  • Model counting.  
  • Encourage children to self-correct their counts. | |
| Consider adaptations for children with special needs. | |
| Make number-related games, books, and other materials accessible to preschool children. | |
| Plan group activities focused on counting. | |
### Strand: Number Sense (cont.)

<table>
<thead>
<tr>
<th>Substrand: Understanding Number and Quantity</th>
<th>Substrand: Understanding Number Relationships and Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrate numerals into different areas of the classroom. Discuss numerals in print in a meaningful context.</td>
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<tr>
<td>Expose preschool children to quantities represented in different forms.</td>
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<tr>
<td>Promote use of the subitizing skill.</td>
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</tbody>
</table>
### Strand: Algebra and Functions

<table>
<thead>
<tr>
<th>Substrand: Classification</th>
<th>Substrand: Patterning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organize the classroom into different categorized storage areas to facilitate classification.</td>
<td>Point out patterns in the environment.</td>
</tr>
</tbody>
</table>
| Include materials and objects for sorting in the environment. | Engage preschool children in conversations about patterns.  
  • Say the patterns aloud as a group to build the rhythm of repetition.  
  • Ask questions.  
  • Help children describe patterns and use descriptive words. |
| Identify opportunities for sorting and classifying in everyday routines. | Plan for children at different levels. |
| Recognize sorting in play. | Play with patterns in various formats  
  • Patterns with objects and pictorial designs.  
  • Patterns through movement  
  • Patterns with sounds  
  • Patterns through rhymes and stories |
| Encourage preschool children in conversations about their sorting and classifying.  
  • Ask questions.  
  • Help children label the groups and verbalize their criteria for sorting.  
  • Encourage children to come up with their own criteria for sorting. | |
| Plan opportunities for preschool children to sort and classify.  
  • Plan for children at different levels.  
  • Integrate sorting into children’s current topic of interest and study. | |
# Unit 6 – Mathematics
## Key Topic 3: Getting to Know Interactions and Strategies That Support Mathematics

### Strand: Measurement

<table>
<thead>
<tr>
<th>Substrand: Compare, Order, and Measure Objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Provide opportunities to promote measurement concepts in the environment.</td>
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</table>

<p>| |</p>
<table>
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<tbody>
<tr>
<td>Observe preschool children’s measurement concepts in everyday play and routines.</td>
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</tbody>
</table>

<p>| |</p>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Facilitate and reinforce measurement concepts in everyday play and routines.</td>
</tr>
<tr>
<td>• Build preschool children’s descriptive and comparison vocabulary.</td>
</tr>
<tr>
<td>• Ask questions.</td>
</tr>
<tr>
<td>• Challenge preschool children to use measurement to solve problems.</td>
</tr>
</tbody>
</table>

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<table>
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</thead>
<tbody>
<tr>
<td>Provide opportunities to compare and order objects.</td>
</tr>
</tbody>
</table>

<p>| |</p>
<table>
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<th></th>
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<tbody>
<tr>
<td>Use literature to illustrate measurement concepts.</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td>Provide small-group activities using standard and nonstandard measurement.</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td>Encourage preschool children to estimate measurement.</td>
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</tbody>
</table>

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<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Encourage preschool children to record and document what they have measured.</td>
</tr>
</tbody>
</table>
**Mathematics: Handout 1 – Getting to Know Interactions and Strategies That Support Mathematics**

### Strand: Geometry

<table>
<thead>
<tr>
<th>Substrand: Shapes</th>
<th>Substrand: Positions in Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refer to shapes and encourage the use of shape names in everyday interactions.</td>
<td>Provide materials and equipment to promote spatial sense.</td>
</tr>
<tr>
<td>Engage preschool children in conversations about shapes.</td>
<td>Support preschool children’s spatial sense in everyday interactions.</td>
</tr>
<tr>
<td>• Encourage preschool children to observe and compare shapes.</td>
<td>• Use spatial words and point out spatial relationships.</td>
</tr>
<tr>
<td>• Talk about shapes and discuss their attributes.</td>
<td>• Expand preschool children’s words.</td>
</tr>
<tr>
<td>Provide materials that encourage preschool children to explore and manipulate shapes in space.</td>
<td>Provide preschool children with planned experiences to promote the understanding of spatial sense.</td>
</tr>
<tr>
<td>Include books, games, and other learning materials with shape-related themes in the preschool environment.</td>
<td>• Songs and games</td>
</tr>
<tr>
<td>Provide preschool children with playful opportunities to explore and represent shapes in a variety of ways.</td>
<td>• Literature</td>
</tr>
<tr>
<td>• Play with blocks.</td>
<td>• Construction</td>
</tr>
<tr>
<td>• Match, sort, and classify shapes.</td>
<td></td>
</tr>
<tr>
<td>• Create and represent shapes.</td>
<td></td>
</tr>
<tr>
<td>• Compose and decompose shapes from other shapes.</td>
<td></td>
</tr>
<tr>
<td>Present preschool children with many different examples of a type of shape.</td>
<td></td>
</tr>
</tbody>
</table>
## Unit 6 – Mathematics
### Key Topic 3: Getting to Know Interactions and Strategies That Support Mathematics

### Strand: Mathematical Reasoning

<table>
<thead>
<tr>
<th>Substrand: Promoting Mathematical Reasoning and Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify and create opportunities for mathematical reasoning.</td>
</tr>
<tr>
<td>Pose meaningful questions and challenge preschool children’s thinking.</td>
</tr>
<tr>
<td>Support preschool children in reasoning mathematically.</td>
</tr>
</tbody>
</table>
Interactions and Strategies That Support Mathematics

- Which interactions or strategies did the teacher in the vignette use? How were they used?
- Which interactions or strategies were not observed? How could they be included?
**Interactions and Strategies That Support Mathematics**

- What stands out for you from these lists?
- What similarities do you see among the different substrands?
- Do you see any general categories that the strategies could be grouped in? What are they?

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**Interactions and Strategies That Support Mathematics**

- Why do you think there are so many strategies that relate to teaching mathematics vocabulary and having conversations with children?
- How might you keep and organize these interactions and strategies so they could be a resource for you?
Interactions and Strategies That Support Mathematics

- Context of the activity
- Observations
- Examples of interactions and strategies observed
- Which examples seemed to be intentionally planned?
- Which examples seemed to be teachable moments?

What examples of mathematics interactions or strategies stood out for you?
- Which ones were easier to identify? Which ones were harder? Why?
- Did some strategies or interactions appear more frequently than others? Which ones?
- Did some strategies or interactions appear to be more effective than others? Which ones?
Interactions and Strategies That Support Mathematics

- Were there examples where an interaction or strategy was specifically designed for a child who is an English learner? Which ones?
- Did you see opportunities for strategies or interactions to be used where they were not used?
- What other strategies or interactions might have been used?

Interactions and Strategies That Support Mathematics

- What examples shared by the speaker caught your attention or stood out for you?
- Which ones were similar to those in the *Preschool Curriculum Framework, Volume 1*? Which ones were new to you?
- What did you learn about how the teacher(s) planned what interactions or strategies to use?
- What do you want to remember to use in your current or future work?
Interactions and Strategies That Support Mathematics

- What interactions and strategies stand out for you?
- Which interactions and strategies were the least familiar to you? Were they from primarily one substrand or across several substrands?

What other information do you need to help you better understand the interactions and strategies that were least familiar to you?
- Were there some mathematical-related vocabulary or concepts that were new to you?
- Where might you find resources or support?
What ideas stood out most for you today?
Which ones reinforced what you have already learned or experienced? Which ones gave you a new perspective or insight?
How might you apply a new perspective to your work now or in the future?
What further information or support do you need?
What first step do you need to take?

Interactions and Strategies That Support Mathematics

_The Intentional Teacher_ (Epstein 2007)
“Mathematics and Scientific Inquiry”

- What were some of the key strategies or kinds of strategies suggested?
- How did they compare with the mathematics strategies in the _Preschool Curriculum Framework, Volume 1_? In what way were they similar or different?
Interactions and Strategies That Support Mathematics

- What were some new ideas or insights you gained about the kinds of interactions and strategies to support young children’s mathematical development?
- What could you use from this book/article in your current or future work?
Focus Statement

Students explore methods for approaching universal design, individualized curriculum decisions, and family partnerships in the context of supporting children’s development and learning in mathematics.

Before You Start

As stated in the introductory chapter to the framework, “a fundamental consideration in planning curriculum for individual children is being responsive to the competencies, experiences, interests, and needs each child brings to the preschool classroom” (California Preschool Curriculum Framework, Volume 1, p. 3). This requires being responsive to diverse cultural communities, languages, family structures, abilities, and socioeconomic backgrounds.

In addition to the diversity just described, children enter preschool with a wide range of experiences related to early language and literacy development (PCF, V1, p. 98). Children’s parents may also have a wide range of language and literacy skills in their home language and/or English. Because mathematics vocabulary is an essential component of children’s mathematical development, it is very important for students to learn ways to support young English learners in acquiring mathematics knowledge and skills. Students can find strategies and resources in the English-language development domain in both the California Preschool Learning Foundations, Volume 1 (PLF, V1) and the California Preschool Curriculum Framework, Volume 1 (PCF, V1) and the Preschool English Learners: Principles and Practices to Promote Language, Literacy, and Learning, 2nd edition.

To support these contributions in curriculum planning, this key topic is divided into three subtopics. Each emphasizes features of the curriculum framework that focus on working with each child as an individual: universal design, individualizing curriculum, and forming partnerships with families.

In the first subtopic, universal design, it is suggested that a guest speaker or a panel present to your class on considerations when planning for children with disabilities or special needs. Suggestions for presenters include early childhood special education teachers, speech and language therapists, occupational therapists, assistive
technology specialists, preschool teachers with experience in including children with disabilities in their classes, and parents of children with disabilities or special needs.

Please note that the same active learning segments for the Universal Design, Individualizing, and Family Partnerships subtopics are used in the social-emotional development, language and literacy, and English-language development domains. Slight modifications are made in each domain to reflect its specific content. This should enable instructors to use each key topic individually in each domain or to collapse the subtopics across domains.

Information Delivery

The following content from the *California Preschool Curriculum Framework, Volume 1* (PCF, V1) is referenced in this key topic and may be delivered through lectures and/or assigned readings:

- California’s Preschool Children (pp. 3-5)
- Universal design for learning (p. 13)
- Domain guiding principles for mathematics: “Provide an environment rich in language, and introduce preschool children to the language of mathematics” and “Support English learners in developing mathematical knowledge as they concurrently acquire English” (p. 235)
- Overarching principle: “Individualization of learning includes all children” (pp. 7-8)
- “Partnering with families in curriculum planning” (p. 23)

Point out that Appendix D in the PCF, V1, Resources for Teachers of Children with Disabilities or Other Special Needs, is a resource for the subtopic of universal design (pp. 319-322).

The *Preschool English Learners: Principles and Practices to Promote Language, Literacy, and Learning, 2nd edition* (PEL Resource Guide), and its accompanying instructional guide are also good resources for the second subtopic on individualizing.
Active Learning

**Subtopic 1: Universal Design**

**Getting it started**
Begin by having students read the definition of and key points about universal design for learning described in the framework (PCF, V1, p. 13). Write each approach (i.e., “multiple means of representation,” “multiple means of expression,” and “multiple means of engagement”) at the top of a sheet of chart paper. Because these concepts may be new to your students, it is suggested that you spend some time discussing each one by reviewing the examples provided in the PCF, V1 and providing some additional ones. Also encourage students to share any experiences they have had with adapting curriculum for young children with disabilities or special needs. Ask a few students to write these examples from the PCF, V1 under each heading.

**Keeping it going**
This segment could be done as an out-of-class assignment.

Point out to students that suggestions for supporting children with disabilities and special needs can be found in the environments and materials and interactions and strategies of the PCF, V1.

Have students work in pairs or small groups, and assign each group a different strand or substrand. Ask them to review the environments and materials and interactions and strategies. In Key Topic 3 of this unit, there is a list of the number of interactions and strategies for this domain that could help you assign roughly equal numbers to the pairs or groups. Ask the students to find examples that would be useful in working with children with disabilities or special needs. Point out to students that most of these are useful for all children when applied with reflection and intention.

It is important to explain to students that partnering with early childhood special education staff is an essential part of determining appropriate adaptations for a child with a disability or special need.

**Taking it further**
Have students work individually or in small groups to review their lists of interactions and strategies that could support
children with disabilities or special needs. Have students write each suggestion on a half sheet of paper or large Post-it® note. Their task is to decide if the suggestion falls under “multiple means of representation,” “multiple means of expression,” or “multiple means of engagement.” Remind students that some suggestions may fall under more than one heading.

Then ask students to place the note on the chart paper with the appropriate heading. You could also have students do this by labeling three sheets of paper or three columns on a sheet of paper with the three approaches and writing the suggestions under their chosen heading.

After students have finished posting their suggestions on the appropriate categories, discuss why they made their matches.

**Putting it together**
You may wish to conclude this subtopic by having students discuss the following questions:

- What are some of the key concepts in universal design?
- Which concepts were clear to you? Which concepts were confusing?
- What other questions do you have about universal design related to mathematics?
- What resources do you think would be most helpful to you in your work?

**Another approach**
This method could be done instead of the preceding exercises or in addition to them as an expansion.

Explain to students that they will be hearing from a guest speaker or panel to discuss considerations when planning for children with disabilities or special needs. This is not intended as an in-depth exploration but as an introduction to some of the ways in which curriculum can be made responsive to the needs of all children.

Ask the presenters to address the importance and ways of supporting children’s mathematical development—including having access to experiences, materials, and activities that promote mathematical learning; interacting with their peers and...
adults in the class; and partnering with families and specialists. Provide the presenters with the information on universal design in the PCF, V1 on page 13 and ask them to include examples of how they’ve used some of the strategies suggested. If students are not familiar with special education, ask the presenters to also give a brief overview that includes a summary of the assessment and IEP process; how services are provided; and ways that the special education teachers, therapists, and other specialists can work with teachers in preschool programs to figure out and/or provide the adaptations for each child who has a disability.

Ask the students to note examples of each of three universal design approaches from page 13 of the PCF, V1 that the speaker or panelists describe that support mathematical development.

After the speakers have left, ask students to write any additional suggestions from the presenters on the chart papers or their own lists. Have students respond to the following questions individually or through a class discussion:

- What information from the presenter(s) caught your attention or stood out for you?
- What are you most confident about in supporting the mathematical development of children with disabilities? What concerns you?
- What new or different perspectives do you have? How has this presentation been helpful?
- What do you want to keep in mind when you are planning ways to modify or adapt the learning environment and experiences for children with disabilities to ensure their progress in constructing mathematical knowledge?

**Subtopic 2: Individualizing**

**Getting it started**

Begin this subtopic by reviewing the section in the framework titled “California’s Preschool Children” (PCF, V1, pp. 3-5). Have students find and summarize the key points and share them through a classroom discussion.
**Keeping it going**

Next have students read the sixth overarching principle, “Individualization of learning includes all children” (PCF, V1, pp. 7-8).

Ask students to give examples of differences you might see in different children for each of the characteristics described in the sixth overarching principle: “... temperament, family and cultural experiences, language experiences, personal strengths, interests, abilities, and dispositions...” Ask some students to chart or take notes of the examples.

Then continue the class discussion by reviewing the examples and asking the students to describe possible implications of these individual differences in supporting children’s mathematical development.

**Taking it further**

Assign a substrand to individual or groups of students and have them review the interactions and strategies for the assigned substrand. The students can reference the lists developed in Key Topic 3 of this unit, or you may provide them with a list. As they review the interactions and strategies, they are to note ways that the strategy can help teachers get to know children individually. For example, the strategies “Observe and listen to children’s counts” in the “Understanding Number and Quantity” substrand (PCF, V1, p. 244) and “Engage preschool children in conversations about their sorting and classifying” in the “Classification” substrand (PCF, V1, p. 262) can help teachers learn about individual children’s understanding and use of language as well as their understanding of and skills related to counting and classifying.

After the students have had time to identify examples in their assigned strategies and interactions, ask for some students to share one or two of their ideas with the whole class. Continue the discussion until students have a good sense of how the interactions and strategies can support individualization for children.

**Putting it together**

Conclude this subtopic by having students consider the following questions that could help them get to know the individual characteristics of children who might be in their...
classrooms and how to plan for each child’s continuing progress in mathematical development:

- What kinds of documentation could you use to help understand individual children in terms of their mathematical development?
- What are some ways you could get to know the families of individual children and how they support their children’s mathematical learning?
- How could you learn about the child’s community?
- How could you find out what things a child is interested in that could be used to engage her in mathematical learning experiences?
- What kinds of information about the child could you obtain by observing him during the different parts of the daily routine?
- What are some ways you could determine each child’s strengths related to his/her mathematical development?

**Subtopic 3: Family Partnerships**

**Getting it started**
Point out to students that there is a section entitled “Engaging Families” at the end of each strand (PCF, V1, pp. 257-258, 269-270, 279-280, 288-289, and 294). Ideas to give to families for use at home are listed in each of these sections. Assign groups of students to each strand and have them list a brief summary of each strategy in their strand on pieces of chart paper—two strategies per sheet with blank space between the two strategies or as headings for two columns. They are then to list examples from the framework under each strategy and post their chart sheets on the wall.

**Keeping it going**
Next have the groups move to another sheet of strategies and add other ideas for implementing the strategy. This is similar to a brainstorming activity, so allow about 5 minutes per rotation and continue until all groups have added ideas to all the strategies.
**Taking it further**

Continue this exercise by having students review pages 3-5 of the PCF, V1, “California’s Preschool Children,” and identify examples of how children’s families are diverse.

Ask students to next think about other ways families are unique:

- Family composition in the home including primary caregivers, siblings, other family members
- Length of time in the United States
- Parents’ educational experiences in the U.S. or other countries
- Parents’ literacy skills
- Types of employment and work schedules
- Parenting beliefs and practices

Discuss how each of these characteristics might impact children’s mathematical development.

Next have the students identify any considerations or additional ways to individualize the strategies for families. You could do this as a large group discussion or have the original groups add the considerations and additional strategies for their strand. Each group could then present its ideas.

**Another approach**

Instead of having students brainstorm ways to implement the strategies for engaging families (described in "Keeping it going"), have students interview teachers and parents to identify examples. These interviews could be conducted in different ways:

- Students first ask teachers and parents for ways that they have partnered. Then have students list these examples under the different strategies.
- Assign students different strategies and have them ask teachers and parents for examples for those specific strategies.

**Putting it together**

Conclude this subtopic by having students respond to these
questions:

• What considerations or ideas about the impact and influence of families on children’s mathematical development caught your attention?

• Which ones resonated with you because of your experiences with children and families?

• Which strategies or interactions do you think are most important to include in order to build strong partnerships with families that foster children’s mathematical development?

• What is a first step you would take in building these partnerships?

Reflection

You could have students reflect on this key topic by having them respond to the following questions for each subtopic or for the overall key topic. This could be done through journaling or as a discussion.

• What ideas stood out most for you today?

• Which ones reinforced what you have already learned or experienced? Which ones gave you a new perspective or insight?

• How might you apply a new perspective to your work now or in the future?

• What further information or support do you need?

• What first step do you need to do?

Deeper Understanding

The resources and references on pages 297-299 of the PCF, V1 contain articles related to the three subtopics: mathematics and children who are English learners; mathematics and children with disabilities or special needs; and mathematics and involving parents. Ask students to choose one of the three subtopics and identify the relevant resources and references. They are to write a discussion of the findings from these articles. This assignment could be extended to deeper research if appropriate for your students and particular class.
Universal Design, Individualizing, and Family Partnerships

Universal Design

- Multiple means of representation
- Multiple means of expression
- Multiple means of engagement
Universal Design

- What are some of the key concepts in universal design?
- Which concepts were clear to you? Which concepts were confusing?
- What other questions do you have about universal design related to mathematics?
- What resources do you think would be most helpful to you in your work?

Universal Design

- What information from the presenter(s) caught your attention or stood out for you?
- What are you most confident about in supporting the mathematical development of children with disabilities? What concerns you?
Universal Design

- What new or different perspectives do you have? How has this presentation been helpful?
- What do you want to keep in mind when you are planning ways to modify or adapt the learning environment and experiences for children with disabilities to ensure their progress in constructing mathematical knowledge?

“Individualization of learning includes all children.”
Individualizing

- What kinds of documentation could you use to help understand individual children in terms of their mathematical development?
- What are some ways you could get to know the families of individual children and how they support their children’s mathematical learning?
- How could you learn about the child’s community?

- How could you find out what things a child is interested in that could be used to engage her in mathematical learning experiences?
- What kinds of information about a child could you obtain by observing him during different parts of the daily routine?
- What are some ways you could determine each child’s strengths related to his/her mathematical development?
Family Partnerships

- What considerations or ideas about the impact and influence of families on children’s mathematical development caught your attention?

Which ones resonated with you because of your experiences with children and families?

- Which strategies or interactions do you think are most important to include in order to build strong partnerships with families that foster children’s mathematical development?

- What is a first step you would take in building these partnerships?
• What ideas stood out most for you today?
• Which ones reinforced what you have already learned or experienced? Which ones gave you a new perspective or insight?
• How might you apply a new perspective to your work now or in the future?
• What further information or support do you need?
• What first step do you need to do?